MEASURING THE LIFETIME OF THE MUON

Last Revised September 19, 2006

QUESTION TO BE INVESTIGATED

What is the lifetime τ of a muon?

INTRODUCTION AND THEORY

Muons are a member of a group of particles called *leptons*. Leptons are considered to be fundamental and indivisible particles. In fact, leptons are considered to behave as *true* point particles in all known interactions. (*Leptos* is Greek for small.) There are three known flavors of lepton: the electron, the muon and the tau.

The muon itself has a mass of 105.7 MeV/c^2, carries one unit of “fundamental” charge and, like all leptons, participates in electromagnetic and weak (electroweak) interactions, but not strong interactions. Muons are much more massive than electrons, and they are also unstable. Muons (and their antiparticles) are currently thought to decay exclusively into an electron (or positron) and two neutrinos, as:

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$
$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

These are electroweak decays as can immediately be determined by the fact that two of the decay products are neutrinos.

The most accurately measured value of the muon lifetime is $\tau = 2.19704(4) \times 10^{-6}$s. This is, of course, a mean value based on the measured decay times of many muons. The distribution of muon decay times has the same form as radioactive decays of nuclei and indeed the decays of all unstable elementary particles. With an initial population of N_o muons, one will thus have

$$N(t) = N_0 e^{-t/\tau} \tag{1}$$

remaining muons at time t. Note that this is completely independent of the choice of the start time of the experiment.

The goal of this experiment is to measure the distribution of muon decay times, and from this to determine the associated lifetime τ. So the first question is: where does one find a good source of muons? High-energy cosmic rays (mostly protons from outside our solar system) provide an abundant source of elementary particles as they collide with atoms in the upper atmosphere. Muons are a secondary product of this process. Cosmic ray protons scatter off nucleons in the atmosphere, producing pions (with a lesser contribution from kaon decays), which subsequently decay weakly into muons and neutrinos (nearly 100% of pion decays follow this decay channel). One of the more common of the many pion-producing interactions is given below.

$$p + p \rightarrow p + p + \pi^+ + \pi^-$$
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

Pion-producing collisions usually involve two initial protons or one proton and one neutron. The ratio of positively charged muons to negatively charged muons produced in the upper atmosphere is between 1.2 and 1.3 at sea level. At sea level, the flux is roughly one muon per square centimeter per second and the average energy is in the range of 4 GeV. Experimental muon energies range from 0 GeV to well beyond the TeV scale.

These muons approach the surface of the earth with speeds very close to the speed of light. Because of relativistic time-dilation, the muon "lives" longer in a frame where it is moving quickly. Remember that moving clocks run slow, so if the lifetime of the muon in its rest frame is t, it will live longer in the lab frame according to the formula:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The maximum distance that a non-relativistic muon can travel during its average lifetime is *ct* = 0.66 Km. For a typical muon the Lorentz factor is of the order of 40, and so many energetic muons can reach the earth's surface from the upper atmosphere. Thus, relativistic effects easily explain why atmospherically produced muons often reach the earth's surface. They do lose energy in the process of traversing the atmosphere (about 2 GeV) suggesting that they are produced with energies close to 6 GeV.

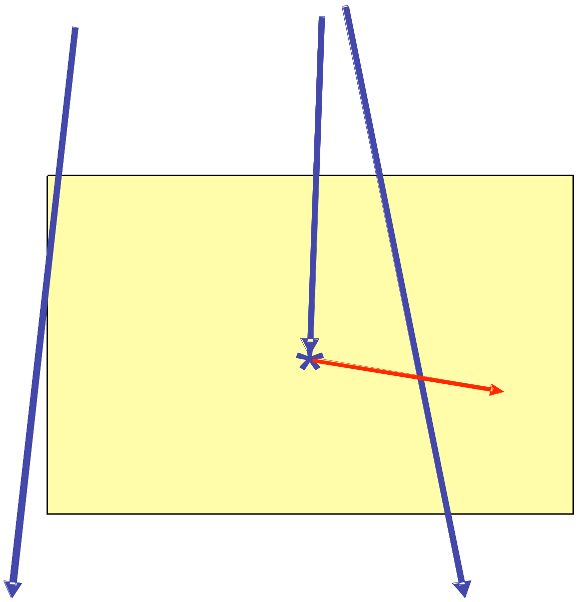

Figure 1. Shown here in blue are the tracks of 3 muons in the scintillator. One of the tracks stops in the scintillator and subsequently decays producing an electron track shown in red.

Detector and Circuit:

How do we detect these muons and the measure their lifetimes? You will be using a plastic scintillator attached to a photomultiplier to detect the light produced by the passage and decay of a muon in the scintillator. The majority of muons entering the detector will be coming down vertically as shown in Figure 1.

A scintillator is a transparent material containing molecules which are excited by the passage of charged particles, and which can produce photons when the molecules subsequently decay. The number of photons produced is proportional (within statistical considerations) to the energy loss of the passing particle. Photons, which happen to be traveling in the right direction, enter the photomultiplier (PMT) to strike a metallic surface (the photo-cathode) and produce free electrons via the photoelectric effect. The electrons are then accelerated in a vacuum by an electric field and strike additional metal surfaces (dynodes) producing additional electrons. Through a series of such stages each initial electron leads to the generation of sufficient numbers of electrons that can easily be measured at the output of the last dynode. Multiple photons will produce a proportionately larger output pulse. Since, the number of photons produced in the scintillator is proportional to the energy loss of each muon, the integrated output charge of the PMT is similarly proportional to that same energy loss. The PMT used in this experiment can detect single photons. To maximize the number of photons that reach the photocathode it is necessary that the scintillator, light guide and photomultiplier tube all be carefully wrapped in reflective aluminum foil. To block external light the assembly is then wrapped in black paper and sealed with black tape. Because of the extreme sensitivity of the PMT it is critical than you always check that no joints have opened up in the outer wrappings.

The detector you will use for this experiment has several parts, but you will only need to use the covering iron plate, the large scintillator and one PMT. A sketch of the detector assembly is shown in Figure 2. The purpose of the iron plate is to reduce the energy of the incoming muons so that more of them may be captured in the scintillator below. You will be using the right photomultiplier tube assembly shown in figure 2.

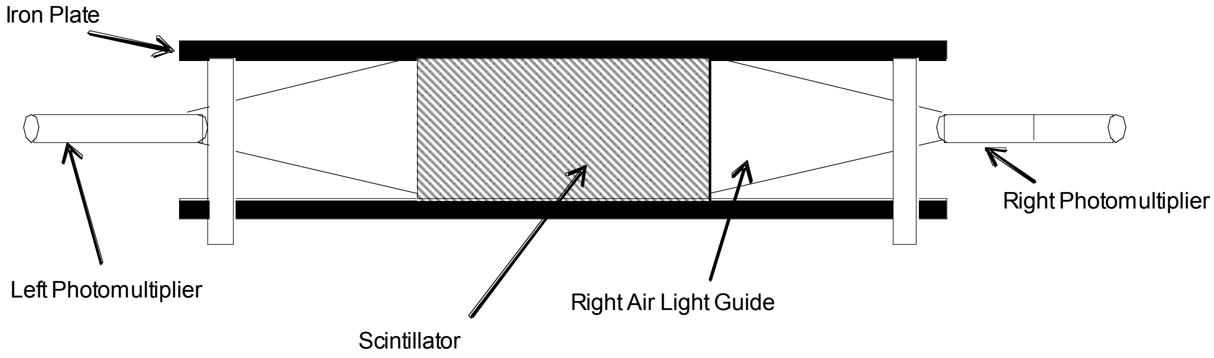

Figure 2. Sketch of the detector.

Because of the many collisions experienced as the muons travel through the scintillator, some muons lose sufficient energy that they slow down to the point that they can be "captured" by one of the atoms in the scintillator. The captured muon will eventually decay producing an electron or positron and two neutrinos. If the electron has sufficient energy it will in turn interact in the scintillator yielding additional photons. Again, the number of photons is proportional to the energy loss of the charged particle, in this case an electron or positron. For the case of the positron, it will in turn interact with a nearby electron annihilating itself and the electron producing additional photons. The decays and subsequent flash of light all occur in a time period of roughly ten nanoseconds. The time between the capture of the muon and its subsequent decay is then determined by measuring the time between the two associated pulses of light.

A circuit has been designed to that will record these two light pulses, and histogram their occurrences indexed by the length of time between them. This circuit is diagramed in Figure 3. When a muon initially enters the scintillator it will release a barrage a photons that in turn produce photoelectrons in the PMT. The PMT cascades this signal in order to create a usable signal that is proportional to the energy of the original photons. The cascade of photoelectrons is accelerated by large electric fields within the PMT; these fields are created by a High Voltage power supply connected

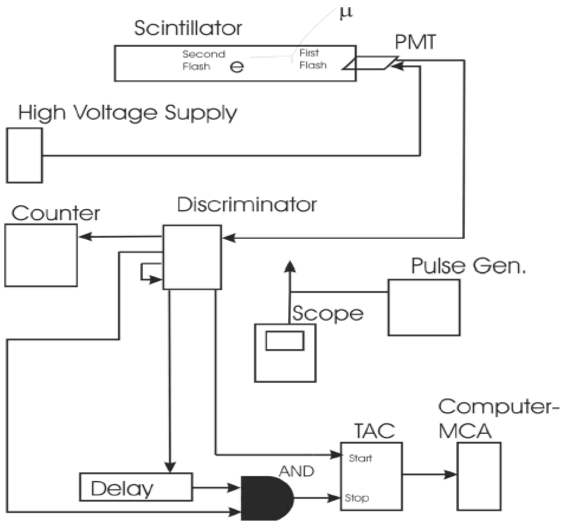

Figure 3. Shown here is the basic circuit diagram used in the experiment.

directly to the PMT. If the PMT signal is large enough it will trigger the Discriminator module, which has a set threshold. This passable event is then simultaneously sent to several places. First, the event is counted by a counter module. Second, it is sent to a Time-to-Amplitude Converter (TAC). This module is started by a signal and begins to ramp up its output as a function of time. The discriminator sends the signal to a delay module as well as an AND gate. The AND Gate is connected to the stopping input of the TAC. When a second signal arrives at the discriminator (which most likely represents the decay of the muon) it is sent to all of the same places, but this time both inputs of the AND gate have the opportunity to be "HI" as the first signal has been delayed. This then stops the TAC and sends a signal to the computer proportional to the length of time between the two signals. The TAC has been tuned to wait signals for times on the order of microseconds, thus signals that are too separated will not register as muon events. The computer will use a Multi-Channel Analyzer (MCA) to histogram the signals from the TAC. (See the website for a more elaborate illustration of the circuit)

Note: Impedence matching is incredibly important to this circuit. All inputs and outputs are to be matched at 50 Ohms. All rack modules are internally matched, but devices like oscilloscopes are not. If you are not observing signals unmatched impedance is most likely the culprit; utilize a 50 Ohm BNC cable.

PROCEDURE:

Finding the "Knee":

Many photon-producing events occur within the detector. Only a few of these events are muon events. In addition to the rather discriminating circuit we have designed, we should also attempt to find the lowest viable setting for the HV bias on the PMT. As we lower the value of the HV bias on the PMT we begin to exclude lower energy events, which effectively reduce noise. Measure the number of events per time interval as a function of HV. Measure the number of events detected at HV settings starting at 1400V and increasing in steps of 50V until you reach 1800V. Each time you change the HV setting you will need to wait about 1 minute for the detector to reach equilibrium. For any results less than 500 counts in a single cycle, repeat the setting until at least 500 counts are achieved or 6 cycles are achieved. Average your results so that all plots are made using the rate per cycle.

When you make a semilog plot of the data in your table you plot should look something like Figure 4 (see Appendix). Room light photons may leak into the detector. Do you notice any difference in your counting when the room lights are on or off?

Figure 4 demonstrates a "knee" at just below 1600V. Above 1600V we see a gradual increase in counts that are a result primarily of non-muon events. Identify the knee from your data and use it this value for taking muon lifetime data later.

Figure 5 (see Appendix) is a plot of the range of muons in various materials. For muons of momenta of 20MeV/c, 50MeV/c, 100MeV/c and 200MeV/c, determine the range that you expect them to travel before stopping. Use the carbon line and determine the value of R at each muon momentum. Assume the scintillator has a density of 1.18gm/cm^3. If c=1 in these units, use dimensional analysis, R, the scintillator's density and the muon's momentum to determine the formula for the distance the muon will travel if it has that initial momentum. From these results, what will be the momentum of a muon just before it enters the detector if it stops at the center? What will be the energy of the muon?

If 1 photon is produced for each 15keV of energy deposited in the detector, how many photons would you expect to reach the PMT? (Hint, remember the detector has two ends!)

Most muons will pass right through the detector. Only a small fraction of the total muon flux will actually stop in the detector. Because of this, there will only be few muon decays per minute in your detector. From the total muon rate it can be shown that the chance of two muons striking the detector within a few microseconds of each other is fairly small (but not zero). Since the time at which a muon strikes the detector is independent of when the previous one occurred at these time scales the time distribution of these "events" is completely flat. This is an important factor in the design of this experiment. Another important factor is the rate at which the PMT emits large numbers of noise pulses. From your "no source" data what HV do you observe high noise rates to occur? What if you were to observe two pulses very close together (Say, for example, a couple of microseconds apart)? By far, the most likely explanation is that you have observed a decay event. The first flash was the arrival of the muon, and the second flash was produced by its decay. (See the discussion in Section 9.4 of Melissinos).

Calibration:

Before beginning the time measurement it is first necessary to calibrate the TAC/MCA system to determine how bin size corresponds to delay time. To do this you will use a pulse generator that can create a pulse train. These pulses can be made to look very similar to the two pulses produced by a muon decay event. Use an oscilloscope to verify that you are getting two pulses from the generator and that they are separated by the amount of time that you want (a range of values between 0.5 and 10 microseconds).

Set the pulse rate to 1-10kHz. By connecting this generator in the place of the PMT you will be able to look at the histogram produced by pulse pairs with an easily measurable separation. By taking a series of such histograms with different (but known) separations you will be able to make a calibration curve such as is shown in Figure 6 (see Appendix). Use the DT cursors on the oscilloscope to measure the time from the start of one pulse and the start of the second pulse. Move the pulses horizontally on the scope (without changing the spacing!) and re-measure the separation. Do this at least 4 times. Use the results to determine the uncertainty in your pulse separations. Feed the double pulse into the discriminator in place of the PMT pulse and start the data acquisition program. You should see a very sharp peak (a "spike") in the histogram. Use the computer cursor to determine the channel number of the "spike". Adjust the pulse separation on the pulse generator to obtain 2 results at low channel numbers and 2 values at high channel numbers. Plot the channel number versus separation and you should see a straight line. From this you can make a linear fit and determine the number of microseconds per bin of your MCA histogram.

Measure the Muon Lifetime

You are now ready to take muon lifetime data! Use the computer to take data at your "knee" for at least one half hour to confirm that you have a roughly exponential distribution as expected in equation (1). Then, starting at the end of a class period, take data for several days. Record each dataset in text format and record the live time in your notebook. Increase the HV by 75 volts and repeat. Take a total of 3 sets of data (3 HV settings). Note that the pulse height distribution that is seen by the MCA in this experiment represents the muon lifetime distribution (a similar curve can be seen in Melissinos Figure 9.27)

DATA ANALYSIS:

Import your data for one HV setting into an Excel spreadsheet. Convert the bin numbers into time using your double pulse calibration results. Is the uncertainty you found using the oscilloscope more or less important than the uncertainty in the bin size? (Hint, how much can you increase the pulse separation and still stay in the same MCA histogram bin?)

Fit the data to determine the muon lifetime. Make the fit using data in the time range 1-11 microseconds. Repeat this for the 3 HV settings. Compare the results and the above uncertainty with the expected muon lifetime. Are your results reasonable?

Some Questions:

1) Does it matter how long it took the muons to reach your laboratory from the upper atmosphere?
2) What kind of effects do you expect from a PMT voltage too high or too low?
3) Your counting data should obey Poisson statistics. This means that the errors are not uniformly distributed on your events vs bin (time) plot. You need to take this into consideration when you do your error analysis.
4) You took data sets for more than one PMT voltage. Do you see any systematic errors in the data?
5) Would one necessarily want a smaller/larger detector than the one in the lab? Which of the two decays (μ^+,μ^-) decays do you think is more likely to be seen in the detector?
7) How fast is the average muon that will stop in the detector?
8) What is the maximum possible energy release in this set-up? How far would that muon travel?

APPENDIX:

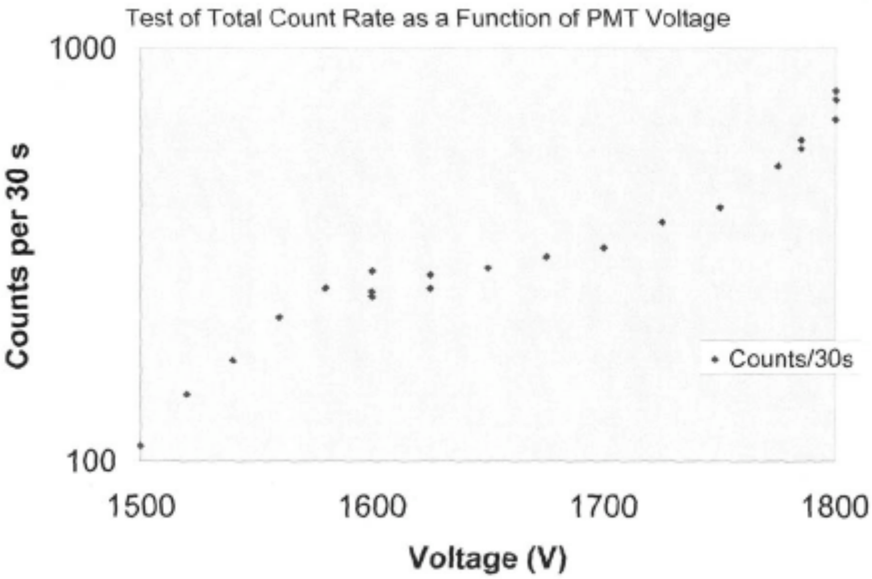

Figure 4. Sample plot of HV scan.

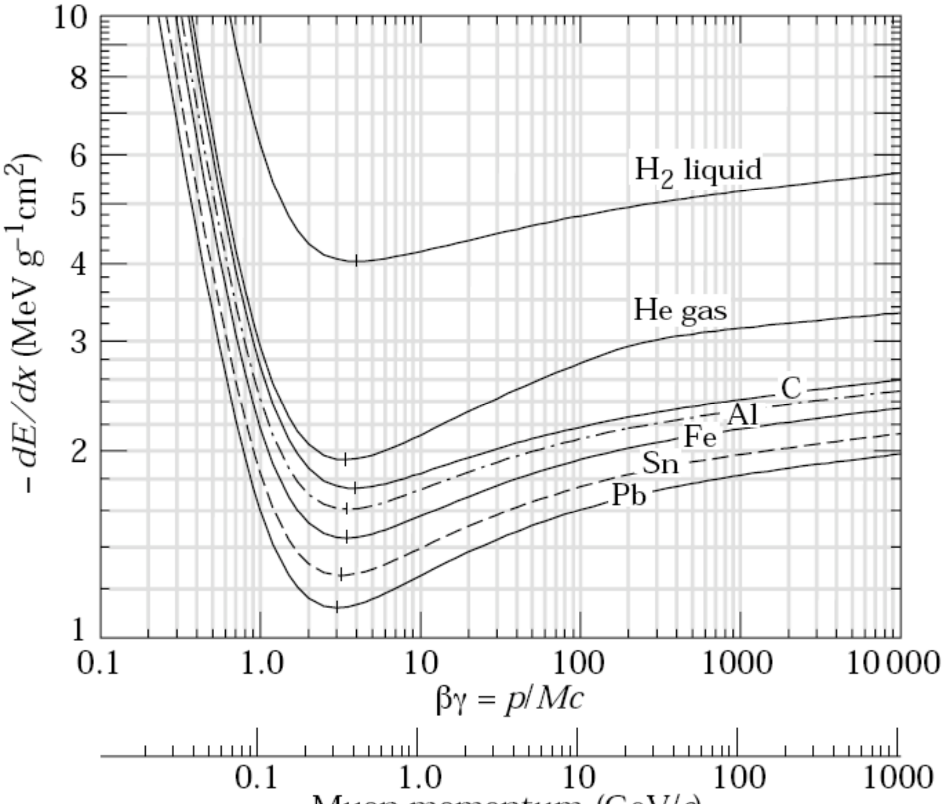

Figure 5: A Plot showing muon energies as they pass through different amounts of different materials. –dE/dx implies the negative change in energy as the particle passes through dx amount of each material.

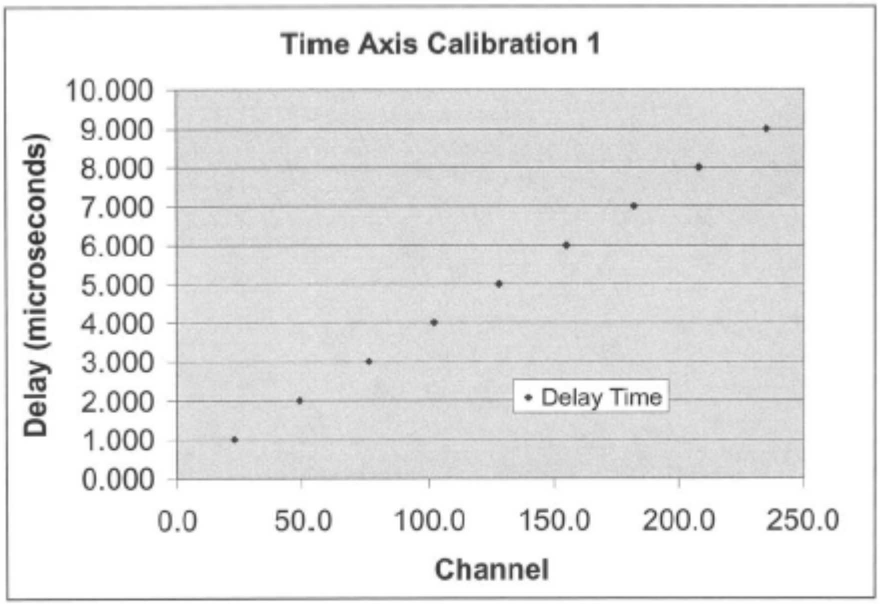

Figure 6. Channel to delay calibration plot.